Accurate Crop Cultivation Date Estimation from MODIS using NDVI Phases and the Extended Kalman Filter

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***Abstract*—Crop yield forecasting is important either for a government, agricultural industries or a trading company for their action plans. A very important variable to be given to a crop model used for the forecast is an accurate crop cultivation date. When the area of interest is large, it is preferable to use remote sensing data such as satellite images for the crop monitoring. The estimation of the cultivation date can be done using the Normalized Difference Vegetation Index (NDVI) collected from the Moderate Resolution Imaging Spectroradiometer (MODIS) aboard the Terra satellite which is interesting for a developing country thanks to its free and frequent availability. However, in a tropical area the NDVI data is very noisy due to clouds. To cope with this issue, it is proposed in the literature that one can model the NDVI by a triply modulated cosine function with the mean, the amplitude and the initial phase as state variables and then use the Extended Kalman Filter (EKF) to estimate these three variables. However, the EKF tuning and cultivation date computation methods in the literature cannot be used in reality for accurate age estimation. This paper therefore proposes to use the estimated total phase of the triply modulated cosine function to compute crop cultivation date. To specify, the crop cultivation date is defined as the day in which the estimated total phase is higher than a threshold. The threshold is predefined using available real cultivation dates from available ground truths during the same year in the region. Thanks to the method proposed in this paper, the mean cultivation date error is reduced from 11.70 days to 0.0 day and the mean absolute error is reduced from 26.2 days to 16.3 days compared to when the method in the literature is implemented for our study cases which are single crop rice fields in the northeast of Thailand.**

I. Introduction

Crop yield forecasting is important either for a government, agricultural industries or a trading company for their action plans. A very important variable to be given to a crop model used for the forecast is an accurate crop cultivation date.

Accurate crop cultivation date estimation is indispensable for crop yield forecasting which is one of major interests of government, agricultural industries or agricultural commodities trading companies. The age of the crop can also be used for forecasting parasite attack trends and for the calculation of suitable government aids when citizens' fields are affected by natural disasters. A technique that enables the crop cultivation date estimation in wide-area is the satellite remote sensing technique. One of a good candidate to be the input for the crop cultivation date estimation is the Vegetation Index (VI) data from the Moderate Resolution Imaging Spectroradiometer (MODIS) which is available for free and has a global coverage.

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The MODIS is boarded on the satellites Terra and Aqua which have both a daily-repeated cycle. Terra passes from north to south across the equator in the morning, while Aqua passes south to north over the equator in the afternoon. Despite the daily-repeated cycle of Terra and Aqua, the MODIS data could be unavailable due to the presence of clouds and off-nadir sensor views. The data from Terra and Aqua is therefore processed independently at an 8-day interval. The data from the day with the least cloud and aerosol contamination and the lowest-view angle, is used to represent all the 8 days in the interval and is called the 8-day composite. Only the data from Terra is used in this studies by assuming that the data from Aqua is more noisy due to higher cloud density in the afternoon and that the NDVI does not change much during one day.

The vegetation indices (VI) are derived by examining spectral reflectance signatures given from leaves. The greener the plants are, the less reflected energy in the visible is due to high absorption by photosynthesis of the leaves. Among the visible bands, the blue band (470 nm) is the most absorbed and the red (670 nm) is the least absorbed. Nearly all of the near-infrared radiation (NIR) is scattered (reflected and transmitted) with very little absorption [Solano et al., 2010]. As a result, the greener the plants are, the higher the contrast between red and near-infrared responses. Two types of Vegetation Index from MODIS are available: the Enhanced Vegetation Index (EVI) and the Normalized Difference Vegetation Index (NDVI). The EVI, taking into account the NIR, the red and the blue bands has a greater sensitivity than that of the NDVI in high biomass area. Furthermore, the EVI is less affected by the atmospheric than the NDVI since it uses the difference in blue and red reflectances as an estimator of the atmosphere influence level. The EVI from MODIS has been used for the estimation of the number of crop cycles in China where the overall accuracy is shown to be of 91% [Li et al., 2014]. However, the spatial resolution of the EVI is at 500 meters which is too coarse to be used for most rice fields in Thailand where there are many small rice producers. The NDVI which is a normalized transform of the NIR to red reflectance ratio and has a spatial resolution of 250 meters will therefore be used in our studies. The NDVI varies from -1 to 1: 1 represents full vegetation and 0 represents no vegetation. However, the problem of the NDVI data from MODIS is that it is very noisy for a tropical study area such as Thailand due to high density of clouds. An efficient estimation method must therefore be chosen to cope with this problem. In [Chu et al., 2014], the NDVI from MODIS was used for the phenology detection of the winter wheat in the Yellow river delta. The Savitzky-Golay filter was selected to denoise the NDVI data. The ground truth of the phenology was not available in [Chu et al., 2014], hence the estimation error of their method is unknown.

Rice cultivation date estimation in Thailand using NDVI data from MODIS has been proposed in [Chumkesornkulkit et al., 2013] where a triply-modulated cosine function was used to model the measured NDVI data and the Extended Kalman Filter (EKF) was chosen to estimate the mean, amplitude and phase parameters of the cosine function. The cultivation dates were estimated as the date where the estimated seasonal variation of NDVI, which is the estimated NDVI minus the estimated mean, is greater than an arbitrary threshold. The estimated cultivation date of single crop rice was compared to the ground truths from the rice department of Thailand in 40 study areas. The mean error of the cultivation date estimates of single crop rice was shown to be of 17 days [Suwannachatkul et al., 2014]. However, to achieve this accuracy, the brute force algorithm where large numbers of tuning parameters of the EKF and threshold were tested until the smallest mean error was obtained. The tuning parameters obtained by this method may therefore suit only for the study areas in consideration and can provide high errors when it is applied to other areas in which the ground truth is not available. Furthermore, the appropriate standard deviation of the NDVI measurement noise in [Chumkesornkulkit et al., 2013] was found to be 20 while the range of the NDVI is between -1 and 1 which is not possible physically.

In this paper, we propose first a tuning method for the EKF to replace the brute force algorithm in [Chumkesornkulkit et al., 2013] with the standard deviation of the measurement noise which is possible physically. Then, we propose a novel method for the cultivation date estimation by using the estimated total phase of the triply modulated cosine function of the NDVI. The paper is organized as follow. In section II, we describe the NDVI from MODIS, the state and the measurement equations and the estimated cultivation date computation method. In section III, the EKF equations and the methods for the filter initialization and parameter tuning are provided. In section IV, we present the results and the analysis where the estimated cultivation dates are compared to the real cultivation dates provided by the ground truths from the rice department of Thailand. The paper ends with concluding remarks.

II. NDVI, State and Measurement Definitions

A. NDVI Measurements

1) 8-day composite NDVI Data from MODIS: As in [Chumkesornkulkit et al., 2013], the 8-day MODIS/Terra data (MOD09Q1) from NASA for the period from the 18th of February 2000 until November 2012 is used for the studies. The NDVI is a normalized transform of the NIR reflectance ρ_{NIR} to red reflectance ρ_{red} ratio, designed to standardize VI values to between 1 and +1. It is commonly expressed as [Solano et al., 2010]:

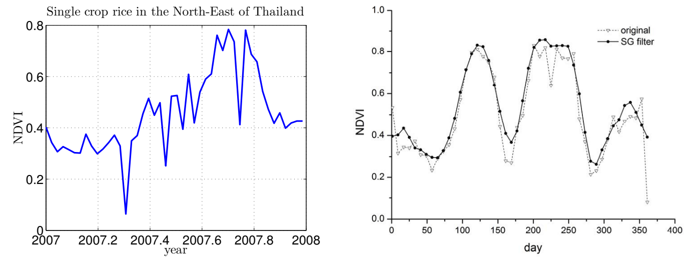

Fig. 1: Example of NDVI data from MODIS produced by single crop rice in Thailand in 2007 (left) and that by winter wheat in the Yellow river delta between 2012 and 2013 in China from [Chu et al., 2014] (right). We observe the higher noise affecting the NDVI in Thailand compared to China. This is due to the tropical climate of Thailand which induces more clouds.

$$NDVI = \frac{\rho_{NIR} - \rho_{red}}{\rho_{NIR} + \rho_{red}} \tag{1}$$

The greener the plants are, the less reflected energy in the visible is due to high absorption by photosynthesis of the leaves. Therefore, the greener the plants are, the higher the NDVI is. Theoretically, the range of the NDVI is from -1 to 1 where $NDVI = 1$ represents full vegetation and $NDVI = 0$ represents zero vegetation. The NDVI from MODIS has a spatial resolution of 250 meters.

The NDVI data is started to be collected from the 1st of January of each year followed by 8 days interval for each measurement. In this paper, 32 pixels in the northeast of Thailand where the single crop rice (rain-fed) is cultivated, are studied. The NDVI data for each pixel from 18th of February 2000 to the 11th of December 2012 are used, which give the total of 587 NDVI data for each pixel.

In figure 1, we compare the NDVI of single crop rice in Thailand in 2007 to that of winter wheat in the Yellow river delta between 2012 and 2013 in China (right). We observe the higher noise affecting the NDVI in Thailand compared to China. This is due to the higher cloud density in Thailand whose climate is tropical.

2) Noise level on the 8-day composite NDVI Data: To visualize the noise level on the 8-day composite NDVI data, we use the wavelet decomposition function (wavedec) available in MATLAB. Denote N as the total number of the instants. The wavelet decomposition, also called the Discrete Wavelet Transform (DWT), converts the signals, here the 8-day composite NDVI, into one high-pass wavelet coefficient series and one low-pass coefficient series of length $N/2$ each. The Haar wavelet which is equivalent to the Daubechies db1 wavelet is chosen as the mother wavelet.

The high-pass wavelet coefficient series are also reconstructed using the waverec function to visualize the measurement noise level and are shown in Figure 2. We remark that the noise is higher during the raining season. For the sake of simplicity, the measurement noise is chosen equal to 0.3 in this study.

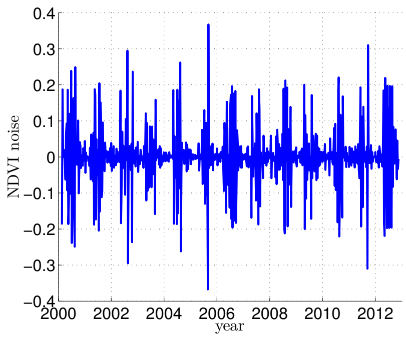

Fig. 2: The noise level on the NDVI data which is the reconstruction from the high-pass coefficient series from the wavelet decomposition.

B. *State and Measurement Definitions*

As in [Chumkesornkulkit et al., 2013], the NDVI is modeled by a triply modulated cosine function. Denote μ_k, α_k and ϕ_k, the mean, the amplitude and the initial phase of the triply modulated cosine function at instant k respectively. Recall that the number of the pixels of interests is 32. The state of the system for the n^{th} pixel x_k^n at instant k is defined as

$$x_k^n = \begin{pmatrix} \mu_k^n & \alpha_k^n & \phi_k^n \end{pmatrix}^T \tag{2}$$

The dynamic of the state is supposed to be governed by a zero mean white Gaussian noise w_k with process noise covariance matrix Q, i.e. $w_k^n \sim \mathcal{N}(0,Q)$, $\forall k \in [1,2,\dots,587]$ and $\forall n \in [1,2,\dots,32]$.

$$x_{k+1}^n = x_k^n + w_k^n \tag{3}$$

In section III-D, we will explain how to choose the process noise covariance matrix Q. The estimated state at instant k for the n^{th} pixel is defined as

$$\hat{x}_k^n = \begin{pmatrix} \hat{\mu}_k^n & \hat{\alpha_k}^n & \hat{\phi}_k^n \end{pmatrix}^T \tag{4}$$

Denote y_k^n the NDVI measurement at instant k for the n^{th} pixel. It is modelled by

$$y_k^n = \mu_k^n + \alpha_k^n cos(\omega k + \phi_k^n) + v_k^n \tag{5}$$

where $\omega = 2\pi f$ is the angular frequency where f is the frequency of the measurement acquisition. For the sake of simplicity $f = 8/365$ is used here seeing that the rain-fed rice has a single crop cycle. v_k^n is the measurement noise at instant k. We assume that v_k^n is a zero mean white Gaussian noise with standard deviation of σ_v, i.e. $v_k^n \sim \mathcal{N}(0, R = \sigma_v^2)$, $\forall k \in [1,2,\dots,587]$ and $\forall n \in [1,2,\dots,32]$. R is the measurement noise covariance matrix. In section III-E, we will explain how to choose σ_v.

The estimated NDVI at instant k for the n^{th} pixel $\hat{y}_k^n$ is calculated from the estimated state $\hat{x}_k^n$ as

$$\hat{y}_k^n = \hat{\mu}_k^n + \hat{\alpha}_k^n cos(\omega k^n + \hat{\phi}_k^n) \tag{6}$$

Denote $\hat{\psi}_k^n = \omega k^n + \hat{\phi}_k^n$ the estimated total phase of the NDVI at instant k for the n^{th} pixel.

III. The Extended Kalman Filter: Equations, Tuning and Initialization

The Extended Kalman Filter (EKF) makes use of the formulations of the Kalman Filter (KF) which is the best linear estimator when process noise w_k and measurement noise v_k are uncorrelated and white even though they are non Gaussian [Simon, 2010]. The KF is the best estimator in the sense that it gives the minimum achievable error variance for a linear system. The proof of the KF is provided in [Kalman, 1960].

The EKF generates estimates for nonlinear systems by linearizing the system at the current estimate and then applies the Kalman Filter equations to the linearized system.

A. *EKF Equations*

Denote $\hat{x}_k^-$ the a priori estimate at instant k, P_k^- the a priori estimation error covariance matrix, $\hat{x}_k$ the a posteriori estimate and P_k the a posteriori error covariance matrix. The algorithm of the EKF can be divided into two stages: prediction and update as follow [Suwantong, 2014]:

Prediction:

$$\hat{x}_k^- = f(\hat{x}_{k-1}) \tag{7}$$
$$P_k^- = Q_{k-1} + \hat{F}_{k-1} P_{k-1} \hat{F}_{k-1}^T \tag{8}$$

$\hat{F}_{k-1}$ is the Jacobian of f evaluated at $\hat{x}_{k-1}$, i.e.

$$\hat{F}_{k-1} = \left.\frac{\partial f}{\partial x}\right|_{x=\hat{x}_{k-1}} \tag{9}$$

Update:

$$\hat{x}_k = \hat{x}_k^- + K_k(y_k - h(\hat{x}_k)) \tag{10}$$
$$P_k = P_k^- - K_k S_k K_k^T \tag{11}$$

where

$$S_k = \hat{H}_k P_k^- \hat{H}_k^T + R_k \tag{12}$$
$$K_k = P_k^- \hat{H}_k^T S_k^{-1} \tag{13}$$

$\hat{H}_k$ is the Jacobian of h evaluated at $\hat{x}_k^-$, i.e.

$$\hat{H}_k = \left.\frac{\partial h}{\partial x}\right|_{x=\hat{x}_k} \tag{14}$$

B. *A priori Initial State*

Denote the a priori initial estimated state for the n^{th} pixel as $\hat{x}_0^{n-} = \begin{pmatrix} \hat{\mu}_0^{n-} & \hat{\alpha}_0^{n-} & \hat{\phi}_0^{n-} \end{pmatrix}^T$.

As in [Chumkesornkulkit et al., 2013], the a priori initial estimated mean $\hat{\mu}_0^{n-}$ and amplitude $\hat{\alpha}_0^{n-}$ for the n^{th} pixel are computed using the NDVI measurements y_k^n, $\forall k \in [0,1,\dots,586]$, i.e. for the n^{th} pixel

$$\hat{\mu}_0^{n-} = \sum_{k=0}^{N} \frac{y_k^n}{N} \tag{15}$$

$$\hat{\alpha}_0^{n-} = \frac{\max_k y_k^n - \min_k y_k^n}{2} \tag{16}$$

The a priori initial estimated $\hat{\phi}_0^{n-} = 120°$ is chosen $\forall n \in [1,\dots,32]$ supposing that the start date which is on the 18th of February 2000 represents the stage after the rice harvest.

C. A priori Initial Error Covariance Matrix

For the sake of simplicity, the a priori initial estimation error $\tilde{x}_0^{n-} = \hat{x}_0^{n-} - x_0^n$ is modelled by a zero-mean white Gaussian noise $\tilde{x}_0^{n-} \sim \mathcal{N}(0, P_1^-)$, $\forall n \in [1,\dots,32]$. P_0^- is called the a priori initial error covariance matrix. Each component of the a priori initial estimate is supposed to be uncorrelated, hence P_0^- is diagonal. High values of the standard deviation of the a priori initial estimation error of each component are chosen to let the EKF explore large domain of possible values until it finds ones that correspond to the measurements the most.

$$P_0^- = \begin{pmatrix} 1^2 & 0 & 0 \\ 0 & 1^2 & 0 \\ 0 & 0 & (\frac{10}{365} \cdot 2\pi)^2 \end{pmatrix} \tag{17}$$

D. Process Noise Covariance Matrix

We suppose that the process noise of each component is uncorrelated. Denote σ_{w_μ} the standard deviation of the noise on the mean, σ_{w_α} the standard deviation of the noise on the amplitude and σ_{w_ϕ} the standard deviation of the noise on the initial phase. The process noise covariance matrix for the n^{th} pixel is therefore described as

$$Q^n = diag\left((\sigma_{w_\mu}^n)^2 \quad (\sigma_{w_\alpha}^n)^2 \quad (\sigma_{w_\phi}^n)^2\right) \tag{18}$$

$\sigma_{w_\mu} = 0.02 \cdot \hat{\mu}_0^{n-}$ and $\sigma_{w_\alpha}^n = 0.05 \cdot \hat{\alpha}_0^{n-}$ are chosen in the study. The standard deviation of the noise on the a priori estimate of the initial phase is chosen to be equivalent to two days, i.e. $\sigma_{w_\phi}^n = (2/365) \cdot 2\pi$ in radian, $\forall n \in [1,\dots,32]$.

E. Measurement Noise Covariance Matrix

We observe in figure 2 that the noise level depends on the season. The noise is high during the middle of the year which corresponds to the raining season where there are lots of clouds. For the sake of simplicity, a constant value of the standard deviation of the measurement noise $\sigma_v = 0.3^2$ is chosen.

IV. Result Analysis

A. Time Vector

Before plotting the estimated states and measurements, we define the year vector t_{year} which is constructed with the same number of instants than the number of the measurements for a pixel, i.e. 587 instants. The start date is on the 18th of February. Therefore, the value of the first component of the year vector $t_{year}(0) = 2000 + (31+18)/366$ since 2000 is a leap year. When the NDVI measurements are collected all year long, the total number of the measurement for the year will be 46. The start date of the NDVI acquisition from MODIS is on the 1st of January of each year.

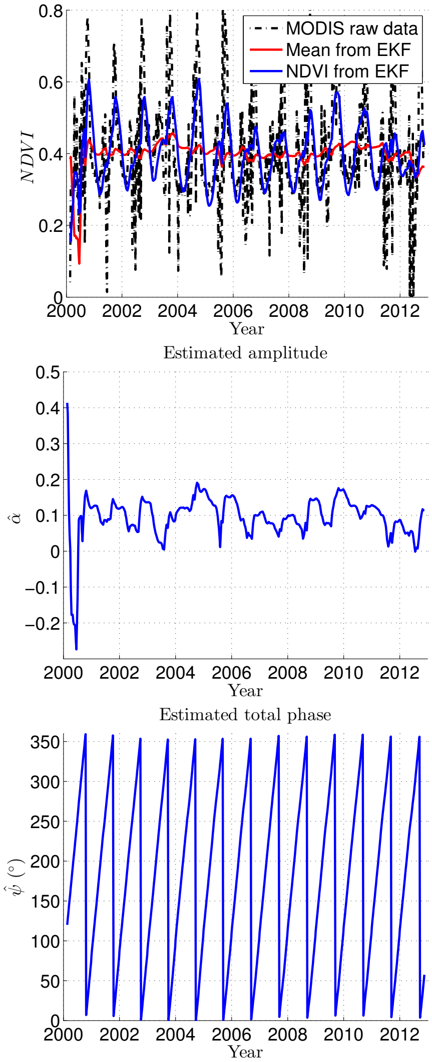

Fig. 3: Examples of the estimated NDVI $\hat{y}$ and the estimated mean $\hat{\mu}$, amplitude $\hat{\alpha}$ and total phase $\hat{\psi}$ in time using the EKF.

B. Estimated State and Estimated NDVI

Examples of the estimated mean μ, the estimated amplitude α, the estimated total phase ψ and the estimated NDVI $\hat{y}$ are shown in figure 3. We observe high variation of the estimated mean and amplitude, which depend on the season during the year. The estimated total phase $\hat{\psi}$ shows the fact that the rice has a single crop cycle very well.

C. Estimated Cultivation Date Computation using Estimated Total Phase

In [Chumkesornkulkit et al., 2013], the estimated mean of the triply modulated cosine function μ_k^n was considered to represent long term vegetation changes such as deforestation and climate changes. Only the term $\alpha_k^n cos(\omega k + \phi_k^n)$, called the seasonal term was considered to be related to the growth of the rice. The estimated cultivation date was set to be equal to the date for which the seasonal term is greater than a predefined

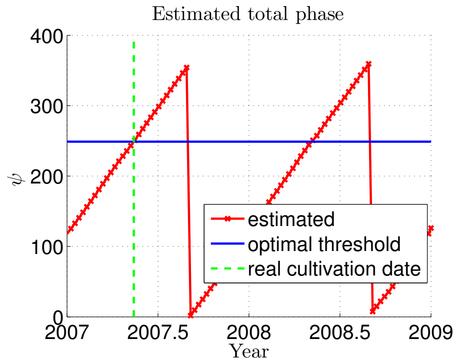

Fig. 4: The optimal threshold for each estimator for each pixel is defined as the y-component of the intersection between the phase of the real cultivation date in green and the estimated total phase in red (right).

Estimator	mean($\tilde{t}$)	std($\tilde{t}$)	mean($\lvert\tilde{t}\rvert$)
EKF as in [Chumkesornkulkit et al., 2013]	11.70	27.41	26.20
EKF as in this paper	-0.04	19.20	16.31

TABLE I: Cultivation date estimation errors in mean, standard deviation, and mean of the absolute error by the EKF using the method in [Chumkesornkulkit et al., 2013] and using the method proposed in this paper.

threshold τ. The minimum mean estimation error was achieved when the threshold $\tau = -0.02$ was chosen for the single crop rice. A negative threshold may represent when the farmer cut off other crops on the field, before the rice cultivation.

In this paper, we propose consider the estimated cultivation date to be the date where the estimated total phase $\hat{\psi}_k^n = \omega k^n + \hat{\phi}_k^n$ of the triply modulated cosine function is equal to a predefined threshold. This is as we use the stage of the crop compared to the entire cycle as its age.

To compute the predefined total phase threshold, we compute the threshold giving zero cultivation date estimation, called here as the optimal threshold for each pixel. Since in reality the optimal threshold is not available for the pixel where cultivation date has to be estimated, we propose that for each pixel, the threshold is equal to the mean of all the optimal thresholds from every other pixels in the same year.

Define $\tilde{t}_k^n$ as the cultivation date estimation error of the n^{th} pixel at instant k using the real cultivation date from the available ground truths as the reference. The mean, the standard deviation and the mean of the absolute error of the cultivation date estimates from the method proposed in this paper and from the method propose in [Chumkesornkulkit et al., 2013] are presented on table I. We remark that the mean error is reduced from 7.89 days to -0.12 day, the standard deviation of the error is reduced from 27.68 days to 18.44 days and the mean absolute error is reduced from 24.94 days to 15.70 days.

V. Conclusions and Perspectives

In this paper, we propose a method for estimating the cultivation date of single crop rice using 8-day composite normalized difference vegetation index (NDVI) from the Moderate Resolution Imaging Spectroradiometer (MODIS). The studied areas are in Thailand, whose climate is tropical, and hence the measured NDVI data from MODIS in this area is very noisy due to high cloud density. A good estimation technique is therefore needed to cope with this high measurement noise. In this study, the NDVI data from early 2000 until late 2012 are used.

The NDVI is modelled by a triply modulated cosine function with the mean, the amplitude and the initial phase as the state of the system which is dynamic. We use the NDVI measurements as inputs for the Extended Kalman Filter to estimate the state of the system. The dynamics of the state is governed by a zero-mean white Gaussian noise. Contrary to the existing work in [Chumkesornkulkit et al., 2013] where the brute force algorithm where large numbers of tuning parameters of the EKF and threshold were tested until the smallest mean error was obtained, we propose a method for tuning the parameters of the EKF. A formulation for the a priori initial error covariance matrix is also provided in this paper. The estimated cultivation dates are compared to the real cultivation dates collected by the rice government of Thailand which are in 2004 and 2007. There are 32 study areas (pixels) in total.

The standard deviation of the measurement noise is chosen by converting the NDVI measurements using the wavelet decomposition to filter the NDVI into the high-pass part, which represents the noise, and the low-pass part, which represents the signal. We then reconstructed the high-pass part to visualize the noise level. In this study, we choose the standard deviation of the measurement noise to be 0.3 which is realistic compared to the range of the NDVI which is between 0 and 1 unlike in [Chumkesornkulkit et al., 2013] where it was chosen to be 20.

Also, in this paper we propose to use the estimated total phase of the NDVI to compute the cultivation date instead of using the estimated seasonal NDVI as proposed in the literature. Thanks to this work, the mean cultivation date estimation error is reduced from 7.89 days to -0.12 day, the standard deviation of the error is reduced from 27.68 days to 18.44 days and the mean absolute error is reduced from 24.94 days to 15.70 days.

For future works, the trend of the optimal threshold should be derived using more study areas. Moreover, the period of the single crop rice is set to be equal to 365 in this study. One could also consider the period of the crop as state variable for further development. Other measurements such as EVI from MODIS, dual polarization image from TerraSAR-X as used in [Vicente-Guijalba et al., 2014] for crop phenology estimation and polarimetric radar data from Radarsat-2 as used in [Vicente-Guijalba et al., 2015] for real time crop phenology estimation should also be combined with the NDVI data to increase the estimated cultivation date accuracy.

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