Optimal Online Station-Keeping Strategy for Communication Satellites in Geosynchronous Orbit

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Abstract— **In this paper, we investigate the algorithm for autonomous station-keeping for communication satellites in geostationary orbit. An impulsive optimal feedback controller, based on the pseudospectral method is proposed. The receding horizon concept is employed to control the satellites toward its predefined slot. Computer simulation results show that the closed-form controller based on state transformation matrix is asymptotically stable with fast error suppression ability, despite the effects from perturbing forces, especially the attractions from the non-spherical Earth, and the third body, i.e. the Sun and the Moon. The fuel expenditure in the proposed algorithm is optimized along the control horizon. Furthermore, it requires less computational burden than general optimization methods, hence it is suitable for online implementation.**

Keywords—Satellite Station-keeping, Impulsive Control, Pseudospectral Method, Receding Horizon Control

I. INTRODUCTION

Traditionally, station-keeping of a communication satellite in geosynchronous orbit is performed using a ground-based, open-loop control strategy. It requires dedicated mission operators to handle such a routine operation. In order to reduce man-power resource, operation time and operation cost, autonomous station-keeping is of high concern. With the advanced propulsion system, such as electrical thruster, equipped on modern communication satellites, automatic low-thrust feedback orbit control can be implemented onboard with high reliability, like the common autonomous system on spacecraft attitude control.

To accomplish such challenging task, the orbital dynamics and control system design is in main focus. The motion under perturbing forces, especially from the gravitational attraction of the non-spherical Earth, and the third body, i.e. the Sun and The moon, makes the control system analysis and design complicated. Many works use truncated version of the geopotential harmonics for the analysis, design and verification of the control systems in a relative coordinate system,. In [1], for example, the mathematical model that includes up to the second geopotential harmonics (J2) has been adopted for the analysis and design and up to J5 for the verification. The control system in [2] succeeds to keep the satellite formation for a long–term operation, under a realistic orbital dynamics. The control strategy, however, is of an open-loop manner. Breger and How [3] employs a model predictive controller to control a satellite relative motion. The control system performs impressively even at a highly elliptic orbit. However, the computation requirement for optimization may be rigorous for autonomous on-line implementation.

In this paper, we investigate for a feedback controller for station-keeping of GEO satellites. Its aim is an algorithm that yields satisfying control performance while fuel expenditure is optimized. Also relatively small computation demand is preferred for practical online implementation. We shall start the investigation by reviewing the equations of motion. The controller design based on a pseudospectral method is then described. The verification of the algorithm and the research conclusion are finally drawn.

II. MOTION DYNAMICS

A. Equations of Motion

Our control system is designed using a coordinate system that the motion states of the satellite are described relatively to its assigned target position. The origin of the coordinate (also known as Hill's coordinate) is at the center of target slot in geostationary orbit, the x-axis is along the radial direction from the Earth's center, the z-axis is along the orbital angular momentum, and the y-axis is defined by the right-hand rule. When the effects from the geopotential harmonics are taken into account, a set of constant-coefficient linear differential equations can be formed as [4]

$$\ddot{x} - 2(nc)\dot{y} - (5c^2 - 2)n^2 x = 0 \tag{1}$$

$$\ddot{y} + 2(nc)\dot{x} = 0 \tag{2}$$

$$\ddot{z} + q^2 z = 2lq\cos(qt + \phi) \tag{3}$$

where $n = \sqrt{\mu / r_C^3}$, μ is the gravitational parameter, r_C is the orbital radius of the reference orbit, R_E is the equatorial radius of the Earth, and c, q and ϕ are the perturbations-related coefficients.

B. State Transition Matrix

State transition matrix can be obtained directly by solving the above differential equations. Alternatively, it can be derived from the difference in orbital elements between the orbits. The advantage of this geometrical approach is that it does not require the tedious solution of differential equations, and it is more accurate than using the Cartesian or curvinear coordinates. Gim and Alfriend [5] have developed an accurate state transition matrix for the perturbed non-circular reference orbit. The transition between the difference orbital elements, $\boldsymbol{\delta e}$, and the relative states in Hill's frame, $\boldsymbol{x(t)}$, is written in the form

$$\boldsymbol{x(t)} = \left\{ \boldsymbol{A(t)} + \alpha \boldsymbol{B(t)} \right\} \boldsymbol{\delta e(t)} \tag{4}$$

or

$$\boldsymbol{x(t)} = \left\{ \boldsymbol{A(t)} + \alpha \boldsymbol{B(t)} \right\} \boldsymbol{D}(t) \overline{\boldsymbol{\varphi}}_{\boldsymbol{e}}(t, t_0) \boldsymbol{D}^{-1}(t_0) \left\{ \boldsymbol{A}(t) + \alpha \boldsymbol{B}(t) \right\}^{-1} \boldsymbol{x}(t_0) \tag{5}$$

where $\alpha = 3J_2R_E^2$. The matrix $\boldsymbol{B}(t)$ contains only the terms perturbed by J_2, whereas $\boldsymbol{A}(t)$ is a linear mapping matrix in the unperturbed case. $\overline{\boldsymbol{\varphi}}_{\boldsymbol{e}}(t)$ is the state transition matrix for the relative mean elements, and $\boldsymbol{D}(t)$ is the Jacobian of the mean to osculating element transformation. Although the derivation of these matrices is complicated, its time-explicit outcome is readily for applications.

III. OPTIMAL FEDBACK CONTROL

General optimal control problem is to find four-tuple $\left\{ \boldsymbol{x}(t), \boldsymbol{u}(t), t_0, t_f \right\}$ to minimize the cost function

$$J = E\left[\boldsymbol{x}(t_f), t_f \right] + \int_{t_0}^{t_f} L\left[\boldsymbol{x}(t), \boldsymbol{u}(t) \right] dt \tag{6}$$

subject to the nonlinear state equations

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \tag{7}$$

the end-point conditions

$$\boldsymbol{e}_{\boldsymbol{L}}^{\boldsymbol{0}} \le \boldsymbol{e}_{\boldsymbol{0}}[\boldsymbol{x}(t_0), t_0] \le \boldsymbol{e}_{\boldsymbol{U}}^{\boldsymbol{0}} \tag{8}$$

$$\boldsymbol{e}_{\boldsymbol{L}}^{\boldsymbol{f}} \le \boldsymbol{e}_{\boldsymbol{f}}[\boldsymbol{x}(t_f), t_f] \le \boldsymbol{e}_{\boldsymbol{U}}^{\boldsymbol{f}} \tag{9}$$

the mixed state-control path constraints

$$\boldsymbol{g}_{\boldsymbol{L}} \le \boldsymbol{g}[\boldsymbol{x}(t), \boldsymbol{u}(t), t] \le \boldsymbol{g}_{\boldsymbol{U}} \tag{10}$$

and box constraints

$$\boldsymbol{x}_{\boldsymbol{L}} \le \boldsymbol{x}(t) \le \boldsymbol{x}_{\boldsymbol{U}} \tag{11}$$

$$\boldsymbol{u}_{\boldsymbol{L}} \le \boldsymbol{u}(t) \le \boldsymbol{u}_{\boldsymbol{U}} \tag{12}$$

where $\boldsymbol{x}$ are the state variables, $\boldsymbol{u}$ are the control input, E is the Mayer component of the cost function, L is the Bolza component of the cost function, $\boldsymbol{e}_{\boldsymbol{L}}^{\boldsymbol{0}}$ and $\boldsymbol{e}_{\boldsymbol{U}}^{\boldsymbol{0}}$ are lower and upper bounds on the initial point conditions, $\boldsymbol{e}_{\boldsymbol{L}}^{\boldsymbol{f}}$ and $\boldsymbol{e}_{\boldsymbol{U}}^{\boldsymbol{f}}$ are lower and upper bounds on the final point conditions, $\boldsymbol{g}_{\boldsymbol{L}}$ and $\boldsymbol{g}_{\boldsymbol{U}}$ are lower and upper bounds on the path constraints.

A. Solving Optimal Control using Pseudospectral Method

The pseudospectral method has been introduced for on-line optimization [6]. It is convenient to divide the time domain into a series of M subdomains so that the time coordinate in each subdomain are

$$t^j \in I^j = [t_0 + (t_f - t_0)(j-1)/M, \quad t_0 + (t_f - t_0)j/M] \tag{13}$$

where $j = 1, 2, ..., M$. Using a Gauss-Lobatto quadrature discretization to approximate the vector field by an Nth degree Lagrange interpolating polynomial:

$$\boldsymbol{f}(t) \approx \boldsymbol{f}^{N_j}(t), \qquad t \in I^j. \tag{14}$$

For optimal interpolation approximation (in the l_2-norm sense), the Lagrange interpolating polynomials are expanded using values of the vector field at a set of Legendre-Gauss-Lobatto (LGL) points. The LGL points are defined on the interval [-1,1] and corresponds to the Zeros of the derivative of the Nth degree Legendre polynomial, $L_N(\tau)$, as well as the end points -1 and 1. The Legendre polynomials are orthogonal to a unit weight function over the interval $\tau \in [-1, \; 1]$. The computational domain is related to the time domain by the transformation

$$t^j \in I^j = \left[(t_f - t_0)\tau^j / M + 2t_0 + (t_f - t_0)(2j-1)/M \right] / 2 \tag{15}$$

where $j = 1, 2, ..., M$.

In each subdomain, there are $N_j + 1$ LGL points. The Lagrange interpolating polynomials take on the following form

$$\boldsymbol{f}^{N_j}(t^j) = \sum_{k=0}^{N} \boldsymbol{f}_{\boldsymbol{k}}^{\boldsymbol{j}} \phi_k^j(\tau^j) \tag{16}$$

where $t^j = t(\tau^j)$ because of the shifted computational domain. The Lagrange polynomials satisfy $\phi_k^j(\tau) = \delta_{ki}$, and hence the coefficients of the polynomial used in the expansion of the vector field take on their values at the LGL point. The

Lagrange polynomials can be expressed in terms of the Legendre polynomial basis functions as

$$\phi_k^j(\tau) = \frac{(\tau^2 - 1)L'_{N_j}(\tau)}{(\tau - \tau_k^j)N_j(N_j + 1)L'_{N_j}(\tau_k^j)} \tag{17}$$

where $k = 0,1,2,...N_j$; $j = 1,2,...,M$.

The vector field consists of values of the states and controls evaluated at the LGL points, and so the states and controls can themselves be considered as approximated via Lagrange polynomials similar to (16). By integrating (16) from the beginning of each subdomain, we obtain

$$\boldsymbol{x_k^j} = \boldsymbol{x_0^j} + \xi^j \sum_{i=0}^{N_j} \phi_i^j(\tau^j) \mathrm{I}_{k-1,i} \boldsymbol{f_i^j},\ k = 1,2,...N_j;\ j = 1,2,...M \tag{18}$$

where $\xi_i^j = dt^j / d\tau^j = (t_f - t_0)/(2M)$, and the entries of the $N \times (N+1)$ integration matrix are defined as $I_{k-1,j} \cong \int_{-1}^{\tau_k} \phi_j(\tau^*) d\tau^*$. The discretization of the cost function is performed using the full quadrature as

$$J^N = E(x_{N_M}^M, t_f) + \sum_{j=1}^{M} \xi^j \sum_{i=0}^{N_j} \boldsymbol{L}(\boldsymbol{x_i^j}, \boldsymbol{u_i^j}, t_i^j) w_i^j \tag{19}$$

where the weights are

$$w_i^j = \frac{2}{N_j(N_j+1)} \frac{1}{[L_{N_j}(\tau_k^j)]^2},\ k = 0,1,2,...,N_j;\ j = 1,2,...,M\ . \tag{20}$$

The optimal control problem is now converted to a simple parameter optimization problem, which can be solved by using any standard procedure with less rigorous computation.

B. Optimal Feedback Controller for Satellite Rendezvous

A simplified version of the general problem where a global quadrature is assumed and constraints are omitted, is applied in our problem. The linear, time-varying relative motion dynamics in Hill's coordinate can be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{B}(t)\boldsymbol{u} \tag{21}$$

Its solution at time *t* can be found in terms of state transition matrix, $\boldsymbol{\Phi}(t, t_0)$, as

$$\boldsymbol{x}(t) = \boldsymbol{\Phi}(t, t_0)\boldsymbol{x_0} + \int_{t_0}^{t} \boldsymbol{\Phi_u}(t, \tau)\boldsymbol{u}(\tau) d\tau \tag{22}$$

where $\boldsymbol{\Phi_u}(t, \tau) = \boldsymbol{\Phi}(t, \tau)\boldsymbol{B}(\tau)$.

When a cost function to be minimized for the optimal fuel expenditure is introduced as

$$J = \frac{1}{2} \int_{t_0}^{t_f} \boldsymbol{u}^T \boldsymbol{u} dt \tag{23}$$

, and subject to the final condition for our regulation problem:

$$\boldsymbol{x}(t_f) = \boldsymbol{x_f} = \boldsymbol{0}\ . \tag{24}$$

The state solution, as well as the cost function at the final time t_f can be calculated at the LGL points as

$$\boldsymbol{x}(t_f) = \boldsymbol{\Phi}(t_f, t_0)\boldsymbol{x_0} + \frac{t_f - t_0}{2} \sum_{k=0}^{N} \boldsymbol{\Phi_u}(t_f, t_k)\boldsymbol{u_k} w_k \tag{25}$$

$$J = \frac{t_f - t_0}{2} \sum_{k=0}^{N} \left(\frac{1}{2} \boldsymbol{u_k^T} \boldsymbol{u_k} w_k \right) \tag{26}$$

Now, the optimization process can be done by minimizing the adjoined cost function

$$\begin{aligned} \overline{J} &= \frac{t_f - t_0}{2} \sum_{k=0}^{N} \frac{1}{2} \boldsymbol{u_k^T} \boldsymbol{u_k} w_k + ... \\ &\boldsymbol{\lambda}^T \left(\boldsymbol{\Phi}(t_f, t_0)\boldsymbol{x_0} + \frac{t_f - t_0}{2} \sum_{k=0}^{N} \boldsymbol{\Phi_u}(t_f, t_k)\boldsymbol{u_k} w_k \right) \end{aligned} \tag{27}$$

,where $\boldsymbol{\lambda}$ is the Lagrange multiplier vector. The problem is now converted into a parameter optimization problem where the solution can be found by taking the partial derivative of (27) with respect to $\boldsymbol{u_k}$ and set equal to zero. The optimal control at each node can be found as

$$\boldsymbol{u_k} = -\boldsymbol{K}(N)\left(\boldsymbol{\Phi}(t_f, t_0)\boldsymbol{x_0}\right) \tag{28}$$

,where the control gain matrix is a function of the number of LGL node:

$$\boldsymbol{K}(N) = \frac{2}{t_f - t_0} \boldsymbol{\Phi_u^T}(t_f, t_k) \left(\sum_{k=0}^{N} \boldsymbol{\Phi_u}(t_f, t_k) \boldsymbol{\Phi_u^T}(t_f, t_k) w_k \right)^{-1} \tag{29}$$

In practice, impulsive control action is commonly applied for satellite orbit control, because it allows post-firing error evaluation for both delta-V magnitude and direction. The impulsive control can be mathematically modeled as

$$\boldsymbol{u}(t) = \delta(t - t_k)\boldsymbol{\Delta v_k} \tag{30}$$

,where $\delta(t-t_k)$ is the Dirac delta function at time t_k, and $\boldsymbol{\Delta v_k}$ is the delta-V vector. The state solution, and the cost function under impulsive control now becomes

$$\boldsymbol{x}(t_f)=\boldsymbol{\Phi}(t_f,t_0)\boldsymbol{x_0}+\sum_{k=0}^{N}\boldsymbol{\Phi_u}(t_f,t_k)\boldsymbol{\Delta v_k} \quad (31)$$

$$J=\frac{1}{2}\sum_{k=0}^{N}\boldsymbol{\Delta v_k^T}\boldsymbol{\Delta v_k}\ . \quad (32)$$

The optimization can be carried out using the same process as the continuous control case, and the impulsive control law can be obtained as

$$\boldsymbol{u_k}=-\boldsymbol{\Phi_u^T}(t_f,t_k)\left(\sum_{k=0}^{N}\boldsymbol{\Phi_u}(t_f,t_k)\boldsymbol{\Phi_u^T}(t_f,t_k)\right)^{-1}\left(\boldsymbol{\Phi}(t_f,t_0)\boldsymbol{x_0}\right). \quad (33)$$

C. Receding Horizon Control

For application to on-line autonomous station-keeping, we apply the receding horizon concept to the impulsive control. The relative states measured at time t_k are used as initial conditions for the optimization, and the optimal control along a pre-defined horizon (t_k, t_k+T), where T is a constant, can be obtained from (33) as

$$\boldsymbol{u}=\left(\boldsymbol{u}_{k/k},\quad \boldsymbol{u}_{k+1/k},\ldots,\boldsymbol{u}_{(k+N)/k}\right)^T \quad (34)$$

Only the first control, $u_{k/k}$, is applied to the system while the others are discarded. The same procedure is repeated until the final time is reached.

IV. Results

In order to verify the proposed control algorithm, a high accuracy relative orbit propagator has been employed. The geopotential harmonics up to 36×36 are included in the dynamics, as well as the effects from the Sun and the Moon. The satellite is aimed to be controlled from a vicinity initial condition to meet its target in geostationary orbit at a mean orbital radius of 42,165 km, and an eccentricity of 0.000553. The control horizon is chosen at one mean orbital period, and there are 16 control nodes. Fig.1 shows the motion of the spacecraft viewed from the target position. It can be seen that our closed-form control solution is asymptotically stable with fast error suppression (within 2.5 orbital periods). The fuel expenditure is optimized along the control horizon, as can be seen from the small delta-V magnitude history in Fig. 2.

V. Conclusion

We have designed and demonstrated a feedback control system for satellite station-keeping. The impulsive form of the state-transition-matrix-based optimal controller can bring and keep the satellite at the target with impressive performance. The simulation results under a high-fidelity orbital dynamics model assure that the algorithm works well in geostationary orbit environment. Moreover, the algorithm requires less computational burden than general optimization methods, hence, it is indeed suitable for on-line implementation.

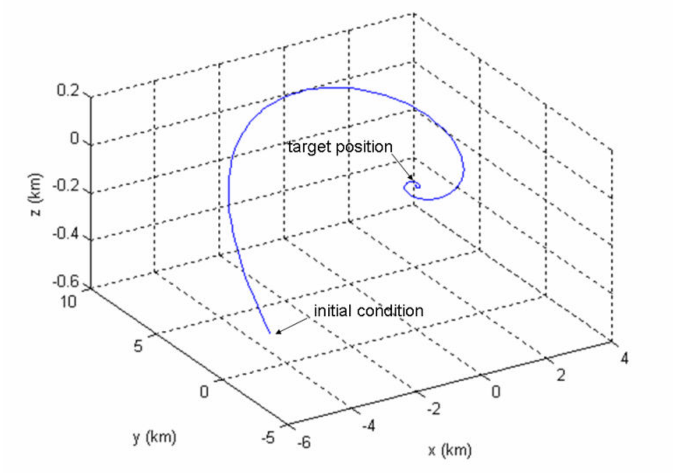

Fig. 1 Relative position trajectory

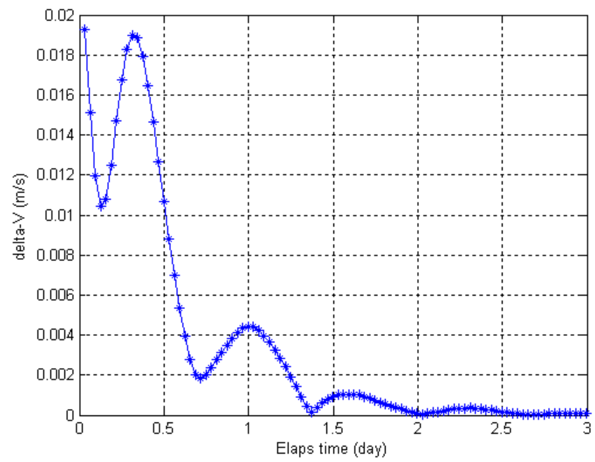

Fig. 2 Delta-V history

References

[1] H. Schaub, S. R. Vadali, J. L. Junkins, and K. T. Alfriend, "Spacecraft Formation Flying Control Using Mean Orbit Elements", Journal of the Astronautical Science, No. 48. No. 1, 2000, pp. 69-87.

[2] S. D'Amico and O. Montenbruck, "Proximity Operations of Formation-Flying Spacecraft Using an Eccentricity/Inclination Vector Separation", *Journal of Guidance, Control, and Dynamics*, Vol. 29, No. 3, 2006, pp. 554-563.

[3] L. Breger and J. P. How, "Gauss's Variational Equation-Based Dynamics and Control for Formation Flying Spacecraft", *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 2, 2007, pp. 437-448.

[4] S. A. Schweighart and R. J. Sedwick, "High-Fidelity Linearized J_2 Model for Satellite Formation Flight", *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 6, 2002, pp. 1073-1080.

[5] D. W. Gim and K. T. Alfriend, "State Transition Matrix or Relative Motion for the Perturbed Noncircular Reference Orbit", *Journal of Guidance, Control, and Dynamics*, vol. 26, No. 6, 2003, pp.956-969.

[6] G. Elnager, M. A. Kazemi and M. Razzaghi, "The Pseudospectral Legendre Method for Discretizing Optimal Control Problem", IEEE Trans. Automatic Control, Vol. 40, No. 10, 1995, pp. 1793-1796.